O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Spring 2001
Final Exam

Name : ______________________________

Student ID: ________________________________

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choose four out of five problems: please indicate as below.

1).____   2).____   3).____   4).____

Problem 1:

Find the *observable* canonical form realization (in minimal order) from SISO continuous-time system given below:

$$\frac{d^4 y(t)}{dt^4} + 3t\frac{d^3 y(t)}{dt^3} + 4\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + \alpha(t)y(t) = \frac{d^2 u(t)}{dt^2} + e^{-t}\frac{du(t)}{dt} + u(t)\ .$$

Notice that gain blocks may be *time* dependent. Show the state space representation and its corresponding simulation diagram.

Problem 2:

There exists a similarity transformation matrix P such that

$$PAP^{-1} = A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{n-1} \end{bmatrix}.$$

Show that if λ is an eigenvalue of the companion matrix A_c, then a corresponding eigenvector is $v = \begin{bmatrix} 1 & \lambda & \cdots & \lambda^{n-1} \end{bmatrix}^T$.

Problem 3:

For the matrices

$$A_1 = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ and } A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

determine the functions of matrices $e^{A_1 t}$, A_2^{99}, $\cos A_2 t$.

Problem 4:
Show that the two linear systems

$$\dot{x}^{(1)}(t) = \begin{bmatrix} 0 & 1 \\ 2-t^2 & 2t \end{bmatrix} x^{(1)}(t) = A_1(t)x^{(1)}(t)$$

and

$$\dot{x}^{(2)}(t) = \begin{bmatrix} t & 1 \\ 1 & t \end{bmatrix} x^{(2)}(t) = A_2(t)x^{(2)}(t)$$

are equivalent state-space representations of the differential equation

$$\ddot{y}(t) - 2t\dot{y}(t) - (2-t^2)y(t) = 0 .$$

a) For which choice is it easier to compute the state transition matrix $\Phi(t,t_0)$? For this case, compute $\Phi(t,0)$.

b) Determine the relation between $x^{(1)}(t)$ and $y(t)$ and between $x^{(2)}(t)$ and $y(t)$.

<u>Problem 5</u>:

Consider the equivalent dynamical equations

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

and

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u$$

$$y = \bar{C}\bar{x}$$

where $\bar{x} = Px$. Their adjoint equations are, respectively,

$$\begin{aligned} \dot{z} &= -A^{*}z + C^{*}u \\ y &= B^{*}z \end{aligned} \qquad (1)$$

and

$$\begin{aligned} \dot{\bar{z}} &= -\bar{A}^{*}\bar{z} + \bar{C}^{*}u \\ y &= \bar{B}^{*}\bar{z} \end{aligned} \qquad (2)$$

where A^{*} and $\bar{A}^{*}$ are the complex conjugate transposes of A and $\bar{A}$, respectively. Show that Equations (1) and (2) are equivalent and they are related by $\bar{z} = \left(P^{-1}\right)^{*} z$.